Efficient SKYCUBE Computation using Bitmaps derived from Indexes

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Abstract

Skyline queries have been increasingly used in multi-criteria decision making and data mining applications. They retrieve a set of interesting points from a potentially large set of data points. A point is said to be interesting if it is as good or better in all dimensions and better in at least one dimension. Skyline Cube (Skycube) consists of skylines of all possible non-empty subsets of a given set of dimensions. In this paper, we propose two algorithms for computing skycube using bitmaps that are derivable from indexes. Point-based skycube algorithm is an improvement over the existing Bitmap algorithm, extended to compute skycube. Point-based algorithm processes one point at a time to check for skylines in all subspaces. Value-based skycube algorithm views points as value combinations and probes entire search space for potential skyline points. It significantly reduces bitmap access for low cardinality dimensions. Our experimental study shows that the two algorithms strictly dominate, or atleast comparable to, the current skycube algorithm in most of the cases, suggesting that such an approach could be an useful addition to the set of skyline query processing techniques.

1. Introduction

Skyline queries are increasingly used in decision support applications such as multi-criteria decision making, data mining, visualization [1] and user-preference queries. A Skyline query over d dimensions returns a set of points that are not dominated by any other point in those dimensions. A point dominates another point if it is as good or better in all dimensions and better in at least one dimension. Consider an example scenario where a person travels to a city and wants to select a hotel to stay by searching the database of hotels in the city. An example dataset is shown in Table 1.

A user who is particular about hotel facilities may issue a skyline query to search for hotels that are close to beach, has pool and gym facilities. The result of this query will include H5, H1 and H3. These hotels are not dominated by other hotels and hence will be interesting to the user. H2 is dominated by H1 as H1 is closer to the beach than H2 and also has gym facilities. Similarly, H4 is dominated by H1 as H1 has gym facilities in addition to being at the same distance from beach as compared to H4. By eliminating hotels that are completely dominated, decision making process can be made much easier for the user.

Table 1. Example of Hotel dataset

	Price	Distance	hasPool	hasGym
H1	30	2	0	1
H2	20	3	0	1
H3	40	4	1	1
H4	10	2	0	0
H5	50	1	1	0

The same dataset, shown in Table 1 may also be subject to other skyline queries. For example, some customers may prefer cheap hotels that are close to beach. They will issue a skyline query for *Price* and *Distance* dimensions. While some others who intend to stay in hotel for a longer period may prefer cheap hotels with excellent facilities and hence may require skyline of *Price*, *hasPool* and *hasGym* dimensions. Depending on the customer preferences, one may expect a skyline query for any subset of dimensions included in the dataset. This is especially true in decision support systems where every parameter is likely to be of interest to some set of users. Algorithms that efficiently calculate Skycube (skyline results for all possible dimension subsets) can be very useful for such applications.

Single skyline algorithms are typically not optimized for Skyline cube computation and have high response times when run multiple times for every dimension subset. The only prior work done in this area was by Yuan et al. [13]. They proposed several computation sharing strategies and proposed two algorithms for computing skycube efficiently. In this paper, we propose two skycube computation algorithms that exploit a bitmap structure to identify whether a point is skyline in any dimension subset. Both the algorithms are completely non-blocking and hence have very low response times.

The first algorithm, called *Point-based skycube*, is an improvement over the single skyline Bitmap algorithm [11] extended to calculate Skycube. In this approach, each record is mapped to an m-bit vector, where m is the sum of the number of distinct values in all dimensions. Once the bitmaps are pre-computed, the algorithm steps through each point to check if it belongs to skyline of any subset of dimensions. It also identifies the duplicates of the current point and masks the points that are dominated by the current point. By identifying duplicates and dominated points (not done in original Bitmap algorithm), the number of points to be processed by the algorithm are significantly reduced.

The second algorithm, called *Value-based Skycube* searches the dimensions for prospective skyline points starting with highest value in all dimensions and stepping down each time the value combination is invalid (no point exists corresponding to the value combination). Once a skyline point is identified, all the value combinations below it are pruned as they are guaranteed to be dominated by the current value combination. The algorithm then repeats the process with next non-dominated value combination. This algorithm is optimized for dimensions with low cardinality where the search space of every value combination of all dimensions is much less than the search space of all points in the database.

Skycube results are typically materialized for faster retrieval. But in some applications where this is not preferred, especially when data is prone to updates, it would be beneficial to materialize some update-friendly intermediate results as this would return results faster than querying base data. Our skycube algorithms are very much applicable in these scenarios. By materializing the bitmaps, which are easily updatable, skycube results can be obtained much faster. This has an added advantage while servicing user-preference queries.

For example, suppose that the user has a preference for a particular set of hotels and would like to know if any of those hotels are a part of final skyline or if there are any hotels that are better than the current choices. While existing algorithms may have to run to completion before they could publish the results, which may take significant amount of time, our Point-based algorithm only needs to access the bitmaps corresponding to the selected hotels to return the results. Another example is where the user enters a *range skyline query* as follows: return the list of skyline hotels with price: 10-30, distance: 2-4 and hasGym. Instead of computing the entire skyline and then filtering the results, the Value-based algorithm can be made to run only for the specified range of values and hence resulting in very low response time. This is especially attractive because datasets used for decision support systems tend to be very large and calculating skycube and then applying the user filter may turn out to be very expensive.

The rest of the paper is organized as follows. In the next section we survey various skyline algorithms proposed in the literature. In Section 3, we explain the Single skyline Bitmap algorithm, which forms the basis for our Point-based algorithm. Section 4 explains how bitmaps can be calculated from database indexes. Sections 5 and 6 explain the Point-based and Value-based skycube algorithms respectively. We provide a thorough experimental evaluation in Section 7 and finally conclude in Section 8.

2. Related Work

This section explains briefly the existing single skyline and skyline subspace algorithms. For the rest of the paper, we shall be considering skylines only for MAX annotations (preferring points with high values in all dimensions) [1], without any loss of generality.

The skyline operator was first introduced by Borzsonyi et al. in [1]. The paper was the first to propose skyline algorithms in database context namely, block nested loops, divide and conquer and B-tree based algorithms. Block Nested Loop (BNL) compares each point with every other point efficiently by keeping a self-organizing list of candidate skyline points in memory. Chomicki et al. proposed an improvement over BNL algorithm by first sorting the dataset according to a monotone sorting function [2]. Divide and Conquer (D&C) algorithm divides the dataset into several partitions, calculates skylines within partitions using a main memory algorithm and merges the result to output the final skyline points. A new D&C algorithm for computing 2D skylines with optimal I/O costs was proposed by Lu et al. in [7].

Tan et al. were the first to propose progressive skyline algorithms based on bitmaps and indexes [11, 3]. Bitmap algorithm is explained in detail in the next section. Index algorithm transforms the points into single dimensional space and organizes them into disjoint lists, each indexed using a B-tree to return skyline points in batches. Nearest Neighbor (NN) was another progressive (online) algorithm proposed by Kossmann et al. in [6]. NN algorithm applies the divide and conquer framework on datasets indexed by R-trees and uses nearest neighbor search techniques [4, 10]. Branch and Bound skyline (BBS) algorithm was proposed in [8] as a progressive and I/O optimal algorithm. It efficiently prunes points by accessing only those R-tree nodes that may contain skyline points.

While all these algorithms compute single skyline efficiently, they are not efficient for calculating skycube because they either 1) use specialized data structures for a particular set of dimensions and extending them for skycube might exponentially increase the number of structures that need to be built on the dataset or 2) are not optimized for calculating skycube as do not adopt any resource or computation sharing strategies.

Recently, lot of research has been going on in the area of skyline subspaces. Yuan et al. were the first to propose algorithms to efficiently compute skycube [13] which consists of skylines of all possible dimension subspaces. Bottom-Up Skycube (BUS) computes skycube in a level-wise, bottom up manner. Each skyline computation uses the results and sorting output of the level below it. The Top-Down Skycube (TDS) extends the basic Divide-and-Conquer algorithm by computing multiple related skyline queries simultaneously. BUS or TDS algorithm adopts several result and computation sharing strategies, while our algorithms reuse the indexes built on the dimensions in addition to sharing computation and bitmap accesses.

Pei et al. introduced the notion of skyline groups and decisive subspaces in [9] and proposed an algorithm, Skyey, to compute subspace skyline points. Tao et al. proposed the SUBSKY algorithm in [12] to efficiently support skyline queries in any subspace. This differs from the skycube algorithms in that SUBSKY aims at computing the skyline of one particular subspace as opposed to all subspaces.

3. Bitmap Algorithm for computing Skylines

This algorithm [11] uses a customized bitmap structure to store all the information required to calculate the skyline points. For example, suppose that we have d dimensional dataset consisting of D points. Each point $x = (x_1, x_2, ..., x_d)$ in the dataset is encoded into an m-bit vector, where m is the total number of distinct values across all dimensions.

- Let k_i be the number of distinct values for dimension $i, 1 <= i <= d$. Then $m = \sum_{i=1}^{d} k_i$. Let p_{ij} be the j^{th} distinct value in dimension i and each $p_{ij} > p_{ij+1}$. Then each p_{ij} is represented by k_i bits where most significant 1 to $j-1$ bits are set to 0 and the rest of the bits are set to 1.

- Each x_i must be equal to some p_{ij} and is mapped to that p_{ij}'s bit vector of length k_i.

Consider an example shown in Table 1(a). The 3-dimensional dataset consists of 3 points. Each of the dimensions have 3, 2 and 2 distinct values and hence every point is mapped to a 7 { 3+2+2 } bit vector as shown. Consider second point $P2\{2, 1, 2\}$. The point has 2nd largest value in dimension d1. Hence it is mapped to bitvector 011. Simiarly it has highest value in 2nd dimension (11), and highest value in third dimension as well(11). Hence the bitmap correspoding to $P2\{2, 1, 2\} = \{011, 11, 11\}$

(a) Data Points

	d1	d2	d3
P1	3	1	1
P2	2	1	2
P3	1	0	1

(b) Bitmap Structure

	d1			d2		d3	
	3	2	1	1	0	2	1
P1	1	1	1	1	1	0	1
P2	0	1	1	1	1	1	1
P3	0	0	1	0	1	0	1

Table 2. Example 1

Let BS_{ij} denote the bit-slice for the j_{th} distinct value of the i_{th} dimension. It corresponds to the column vector in Table 1(b). Number of bits in BS_{ij} will be equal to D, total number of tuples in dataset. A bit that is set in position k will indicate that record Pk has a value of j or higher in dimension i. In our example, $BS_{13} = 100$. This bitmap indicates the records that have a value of 3 or higher in d1 dimension. In our dataset, only $P1$ has a value $\geq$ 3 in d1. Hence the bitmap has 1 for $P1$, 0 for $P2$ and 0 for $P3$.

To check if a point $x = (x_1, x_2, ..., x_d)$ is in the skyline, the following bitmap operations need to be performed:

1. Let A = Bitwise-and of the bit slices corresponding to values x_1, x_2, .. x_d. A bit that is set in position i of the resulting bitmap means that record Pi has values that are good or better than the current point, x, in all dimensions.

2. Let B = Bitwise-or of the bit slices corresponding to just one value higher than each of x_1, x_2, .. x_d. A bit that is set in position i of the resulting bitmap indicates that record P_i has higher value than current point, x, in at least one dimension.

3. Let C = Bitwise-and of results [1] and [2]. A 1 in position i of the resulting bitmap means that record Pi has value that is as good or better than x in all dimensions and better in at least one dimension when compared to x. In other words, P_i dominates x. If on the other hand, the resulting bitmap is all 0's, then no point dominates x and hence x is a skyline point.

Example 1: Referring to the dataset in Table 2, to determine if point $P2(2, 1, 2)$ is in the skyline of all three dimensions, we carry out the above operations:

$A = 110\&110\&010 = 010$
This indicates that no point (other than $P2$ itself) has values that are good or better than $P2$ in all dimensions.

$B = 100|000|000 = 100$
This indicates only $P1$ has value that is stricly better than $P1$ in at least one dimension.

$C = A\&B = 010\&100 = 000$

This indicates there is no point that has equal or higher value than $P2$ in all dimensions *and* strictly higher value in at least one dimension. Hence $P2$ is a skyline point.

3.1. Applicability of Bitmap Algorithm for Skycube computation

There are certain characteristics of the bitmap algorithm that make it very suitable for Skycube computation.

- *Bitmap Reuse*: One of the major limiations of the bitmap algorithm is the cost of accessing the bitmaps. For each point, $2 \times d$ bitmaps have to be accessed where d is the number of dimensions (d for calculating A value and another d accesses for calculating B value). It is worthwhile to note that the bitmaps accessed to check if point P is in skyline for a particular dimension set can be easily reused for checking if the point is skyline in any of its subsets. This reuse of bitmap amortizes their access cost over the computation of skylines for all non-emtpy dimension subsets and hence could lead to better performance.
- *Computation Reuse*: On the lines of the above arguement, if the same set of bitmaps could be used, then the bitwise-and and bitwise-or of those bitmaps can be reused as well.

From above, we can clearly see that the cost of bitmap access for computing single d-dimensional skyline using bitmap approach is same as the cost of bitmap access for computing skycube for all non-empty dimension subsets ($2^d - 1$). We now turn to explain our algorithms for computing skycube using the bitmap structure.

4. Using Indexes to build Bitmap structure

This section explains how to compute bitmaps from database indexes. The bitmap structure computed as explained in the previous section gives information about records that have a value greater than or equal to the current point's value in a specific dimension. This information can be easily extracted from the database indexes, if we assume we have either a Bitmap index or B+ tree index on each of the columns included for skycube computation.

- *Bitmap Index* on a particular dimension maintains a bitmap for each distinct value present in the dimension. Each bit in the bitmap corresponds to a record, and if the bit is set, it means that the record contains the key value. The bitmap structure used in Bitmap skyline algorithm can be built from Bitmap index by doing a Bitwise-OR of bitmaps of all values greather than and including the current value. The resulting bitmaps will have the bits set for all records that have values greater or equal to the key value. Hence by just doing one pass over the bitmap index, the bit structure in Table 1(b) can be easily computed.
- *B+ Tree Index* can also be used to build the bitmap structure in Table 1(b). By scanning the leaf pages of the index from the greatest to the smallest order, one will be able to retrieve the sorted order of the records for the dimension on which the index is built. Once we have a sorted list, building the bitmap structure for that dimension is straightforward.

Many single skyline algorithms require a specific structure, either in R-tree or B+-tree format, for a particular set of dimensions. The disadvantage with this is that these structures are special purpose data structures only used in skyline computation and have very little use outside of skyline applications. Whereas, in a typical decision support system, if any dimension is important enough to be included in skycube computation it is reasonable to assume either B+ tree or Bitmap index to exist on that dimension since they might be needed by lots of other applications as well. Note that even in the absence of the index for some or all dimensions, the bitmap structure can still be computed by sorting the dataset on the non-indexed dimensions. Database indexes are usually optimized for disk acess and would give better performance than any user implemented structures. The bitmap structure can either be pre-computed or computed on the fly.

5. Point-based Skycube Algorithm

Despite the simplicity of the bitmap algorithm presented in Section 3, it has some serious limitations:

- The bit operations have to be performed for every point in the dataset. This could be prohibitively expensive for large datasets. By applying some efficient pruning techniques, many points can be ruled out without accessing their bitmaps.
- The algorithm fails to retrieve the maximum possible information from the bitmap. For example, the algorithm accesses the same set of bitmaps for every duplicate record found in the dataset. If the algorithm were able to identify duplicates, these extra bitmap accesses and subsequent bit operations can be avoided, thus significantly reducing the runtime of the algorithm.

In this section we present our Point-based skycube computation algorithm. It is built on the single skyline Bitmap algorithm explained in Section 3. Point-based algorithm aims to retrieve the maximum possible information from the bitmap by identifying points that are *duplicate* of and *dominated* by the current point. We also propose an heuristic technique to process the points having maximum dominating power first. The following sections explain the steps in detail.

5.1. Point Pruning Techniques

5.1.1 Pruning Duplicate Points

The bit slices accessed by a particular record are determined by the values that the record holds in a particular dimension. Hence every point that has the same value in all or some of the dimensions, will repeatedly access the same set of bitmaps and thereby increasing the number of bitmap accesses. It should also be noted that, if a point is not in the skyline for a particular set of dimensions, then none of its duplicates are skyline points as well and vice versa. By identifying and eliminating duplicates, we not only reduce bitmap access but also converge to result faster as the total number of points that need to be processed by the algorithm are significantly reduced.

Recall the computation of A and C values from Section 3. A bitmap has bit set for those records that have a value higher or equal to the corresponding dimension values in the current record and C bitmap has bit set for records that dominate the current record. The difference (bitwise-xor) between the two bitmaps identifies the records that have the same value as the current record in all the dimensions:

$duplicates = A$ xor C.

Hence by applying the result of current record to all its duplicates, we can avoid processing them separately.

5.1.2 Pruning Dominated Records

For each point in the dataset, the original bitmap algorithm only checks if the current point is dominated by some other point. It would be beneficial to identify and prune points dominated by the current point as well. This can easily be done without any extra computation or extra bit access.

Recall that the B value computed in the bitmap algorithm has bit set for those points that have a value greater than the current point in at least one dimension. That means if a bit is not set for a particular point, then all of its values are either equal or lesser than the current point. In other words, these points are either duplicates of current point in all dimensions or dominated by current point. Since duplicates are handled separately as mentioned above, the points that have bits unset in B bitmap can be pruned from further processing.

5.2. Point-based Skyline Cube Algorithm

The algorithm, listed in Algorithm 1, maintains a Mask (or pruned) bitmap of length D (number of tuples) for every non-empty dimension subset. The bitmaps indicate records that are masked from processing in each of the dimension subsets. The bitmaps are initialized to 1, indicating that no points are pruned in the beginning. The algorithm first calculates all single dimensional skylines. Since, these points have a high value in at least one dimension and are less likely to be dominated by other points, they are the first points to be considered for calculating Skyline cube (shown in Algorithm 2).

Algorithm 1 Point-based Skyline Cube

```
1:  for each non-empty dimension subset $i$ do
2:      initialize the mask bits to 1
3:  end for
4:  for $1 \le j \le d$ do
5:      List L = Points with maximum value in this dimension
6:      for each point P in L do
7:          CheckSkylineCube(P)
8:      end for
9:      clear the mask bits to 0 indicating that the current dimension is completed
10: end for
11: while (List L = getNextPoints()) is not empty do
12:     for each point P in L do
13:         CheckSkylineCube(P)
14:     end for
15: end while
```

After calculating one-dimensional skylines (lines 4-10), the algorithm retrieves the next batch of points to be processed and checks each point for skyline in all dimension subsets (lines 12-14).

Algorithm 2 CheckSkylineCube(Point P)

```
1: for each non-empty non-single dimension subset of P do
2:     if P is not masked in this dimension subset then
3:         check if P is skyline point
4:         apply the result to duplicates of P
5:         mask the points dominated by P for this dimension set in addition to masking P and its duplicates
6:     end if
7: end for
```

The next batch of points to be processed (L = getNextPoints()) can be retrieved in any order. It would not affect the correctness of the algorithm. However, this order has significant impact on the performance of the Point-based algorithm. If a point with high dominating power is processed first, it would prune out more number of points earlier, leading to potentially less number of steps to completion. In order to improve the performance, we propose the following heuristic approach for this ordering:

- The points with maximum dominating power are less likely to be masked in many dimension subsets. Also more points would be masked at lower levels of the lattice (e.g. 1 dimension) than at the upper levels (e.g. d dimensions). Hence, one heuristic technique to retrieve the next batch of points would be to do a bitwise-& of mask bitmaps starting at top level of the lattice, going down one level at a time and stopping just before all points are masked. The points that are still unmasked in this step will be included in the batch for next processing. We have used this technique in our implementation.

6. Value-based Skyline Cube Algorithm

It is quite obvious that since Point-based algorithm processes a single point at a time, the same bit slices may be accessed multiple times. They do not have any specific access pattern and hence caches might not be of much help. As a result, the performance of Point-based algorithm degrades rapidly as the size of the dataset grows (i.e. the number of points to process increases). Consider a case where each dimension has a small number of distinct values and the dataset contains 1 million points. If there are more dimensions, there will be lot more skyline points and the Point-based algorithm might take significantly long time to converge. In order to optimize the bit access, we propose the Value-based Skyline algorithm which exhaustively searches the value combinations for potential skyline points instead of enumerating the points.

6.1. Bitmap Structure for Value-based Algorithm

There is a slight difference in way the bitmaps are calculated for Value-based algorithm. For Point-based skycube algorithm, if p_{ij} is the j^{th} distinct value in dimension i and each $p_{ij} > p_{i(j+1)}$, then each p_{ij} was represented by k_i bits where most significant 1 to $j-1$ bits are set to 0 and the rest of the bits are set to 1.

For Value-based algorithm, if p_{ij} is the j^{th} distinct value in dimension i, then only j^{th} bit of p_{ij} will be set to 1. Rest of the bits will be set to zero. Table 3 shows the bitmap used by Value-based algorithm for our example dataset.

It is quite clear from the table that the bitmap structure required by Value-based algorithm is no different from the conventional bitmap index on each of the dimensions. It means that for the dimensions having bitmap indexes, no other extra data structure would be needed. This is one of the main strengths of this algorithm.

(a) Data Points

	d1	d2	d3
P1	3	1	1
P2	2	1	2
P3	1	0	1

(b) Bitmap structure for Value-based skycube algorithm

	d1			d2		d3	
	3	2	1	1	0	2	1
P1	1	0	0	1	0	0	1
P2	0	1	0	1	0	1	0
P3	0	0	1	0	1	0	1

Table 3. Example 2

6.2. Value-based Skyline Cube Algorithm

The Value-based algorithm, shown in Algorithm 3, starts with highest value combination. For every value combination, it checks if there exists any point corresponding to the current value combination and if there is, the current values are checked for skyline in all dimension subsets. Else, the algorithm retrieves the next interesting point and continues the process until all value combinations are extracted. *checkSkyline()* method is the same as explained in Algorithm 2. The procedure for retrieving the next interesting point, *nextValueExists()*, is explained in the following section.

Algorithm 3 Value-based Skyline Cube

```
1:  for $i = 1$ to $NUM\_DIM$ do
2:      // initial value combination to be checked should be
3:      // the combination with maximum values in all
4:      // dimensions (has maximum dominating power)
5:      values[i] = maxValue[i]
6:  end for
7:  repeat
8:      // isValid returns the first dimension where the value
9:      // combination becomes invalid; returns -1 for valid
10:     // combinations
11:     if (dimToChange = isValid(values)) == -1 then
12:         checkSkyline(values)
13:     end if
14: until nextValueExists(dimToChange, values)
```

Before we delve into the details of the algorithm, we want to review the following examples in order to get some insights.

Example 2: In this example, we explain Value-based skycube algorithm using the dataset shown in Table 2(b).

1. The algorithm starts with a combination having highest value in each dimension. i.e. (3, 1, 2) in our case. 3 is

the highest value in d1 dimension and 1, 2 are highest values in d2 and d3 dimensions respectively.

2. The algorithm then checks if this value combination is valid. This is done by doing a bitwise-& of the bit slices corresponding to the dimension values. If the resulting bitmap is all zeros, then there does not exist any point with this value combination. Hence the value combination becomes invalid. In our case,

   bitwise-& of the bit slices = $100\&110\&010 = 000 \Longrightarrow$ the value combination is invalid.

3. For invalid points, the next logically lower value has to be tried. This is retrieved by lowering the value in the rightmost dimension, if possible, and increasing the value in all subsequent dimensions (if any) to the highest possible value. The next value in our case : (3, 1, 2↓) ⇒ (3, 1, 1)

   bitwise-& of the bit slices = $100\&110\&101 = 100 \Longrightarrow$ The value combination corresponds to point P1 in our dataset.

4. Once the valid value combination is retrieved, check if the value combination is in skyline of any of dimension subsets. In our example (3, 1, 1) is in the skyline of {d1}, {d1, d2}, {d1, d3} and {d1, d2, d3}.

5. Since the value combination was a skyline in {d1, d2, d3}, the next logical value (3, 1, <1) is known to be dominated by the current combination. Hence the algorithm does not consider any points in the pruned space (3, 1, <1). There are no such pruned points in this case.

6. For valid points, the next value combination is got by going to the highest value in the rightmost dimension and going to lower value in the dimension preceding to it. i.e. (3, 1↓, 1↑). This ensures that the next value combination is not in the pruned region. In our examples, our next value would be (3, 0, 2). The algorithm then continues the process from step [2] until the entire search space is exhausted.

6.2.1 Retrieving next value combination

This algorithm has two cases: current value combination is either 1) valid value combination or 2) invalid value combination.

1. Valid value combination: Let's say the current $valid$ value combination corresponds to values $(x_1, x_2, ..., x_d)$. The procedure to retrieve the next interesting point does not depend on whether the value combination corresponds to a d-dimensional skyline point or not. This is because, if the current point is in skyline, then all values combination in the region $(x_1, x_2, ..., < x_d)$ would be dominated by the current point and hence need not be considered. If the current point is not in the skyline, the current value combination $(x_1, x_2, ..., x_d)$ is itself dominated and hence any point in the region $(x_1, x_2, ..., < x_d)$ is guaranteed to be dominated as well.

   The next value combination to be considered is obtained by finding the first i for which $x_i \neq maxValue(i)$[1], $i = d \rightarrow 1$. If $\exists i$, then update $x_i = maxValue(i)$. The algorithm then tries to find the first j for which $x_j \neq minValue(j)$[2], $j = i - 1 \rightarrow 1$. If $\exists j$, then update x_j =next lower value of x_j in dimension j. If no such i or j exists, then all value combinations have been exhausted or dominated and the algorithm is done. Table 4 shows some examples for the same dataset shown in Table 2(a).

Table 4. Next Value for VALID points

Current Value	i	j	Next Value
$(3, 1, 1)$	3	2	$(3, 1\downarrow, 1\uparrow) = (3, 0, 2)$
$(2, 1, 2)$	1	N/A	All remaining values are dominated

2. Invalid value combination: Let's say the current value combination becomes invalid at dimension t. This means that there exists some points with values $(x_1, x_2, .., x_{t-1})$ upto $t - 1$ dimensions but no point matches $(x_1, x_2, .., x_{t-1}, x_t)$ combination in dimension t. The next value combination to be considered is obtained by finding the first i for which $x_i \neq minValue(i)$, starting from $i = t \rightarrow 1$. Update x_i to the next lower value of x_i in dimension i. Then $\forall j = i + 1 \rightarrow d, x_j = maxValue(j)$. If no such i exists, then all value combinations have been exhausted and the algorithm is done. Table 5 shows some examples for the same dataset shown in Table 2(a).

Table 5. Next Value for INVALID points

Current Value	t	i	Next Value
$(3, 1, 2)$	3	3	$(3, 1, 2\downarrow) = (3, 1, 1)$
$(3, 0, 2)$	2	3	$(3\downarrow, 0\uparrow, 2\uparrow) = (2, 1, 2)$

6.2.2 Dealing with columns having high Cardinality

Since the Value-based algorithm exhaustively searches the entire data space for potential skyline points, it performs

[1]maxValue(i) returns the maximum value in dimension i.
[2]minValue(j) returns the minimum value in dimension j.

very well for low number of dimensions and low number of distinct values per dimension. However, it is not uncommon to have some columns with large number of distinct values. In the hotel example shown in Table 1, *Price* is one such attribute. In a dataset of around 1 million points, the number of distinct *price* values would be at least 1000. And this increase in number of distinct values in just one dimension could blow up the size of the search space for the Value-based algorithm.

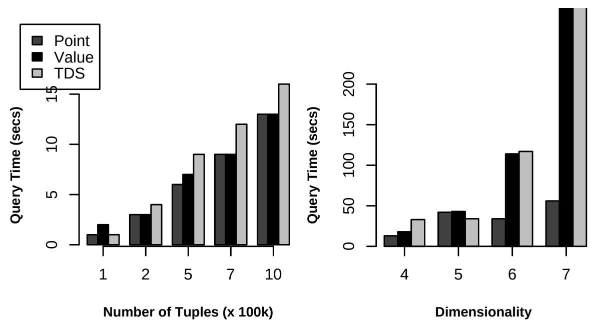
To overcome that, we propose a bucketized version of the Value-based algorithm, where dimensions with high cardinality, will be split into multiple levels, each level containing a specified number of buckets. The number of buckets will basically be the number of distinct values at each level.

While this increases the dimensionality of the dataset as a result of splitting single dimension into multiple levels, the advantage here is that we only need to access lower levels to compare records within the same bucket. With the interest of space, we omit the details of this generalization. Please refer to [5] for full details.

7. Experimental Evaluation

This section provides a thorough performance analysis of our Point-based and Value-based skycube algorithms. In this section in order to validate our algorithms, we compare the performance of our Point-based and Value-based algorithms with the Top-Down skycube (TDS) algorithm proposed in [13]. TDS performs much better than Bottom-Up skycube (BUS) algorithms in almost all cases as shown in [13] and hence we do not consider BUS in our experiments. As mentioned before, single skyline algorithms are not optimized to share computation across dimension subspaces and do not perform well for calculating skycube. As a result we limit our performance evaluation only to the previous skycube algorithm.

All experiments were carried out on a Linux machine having two Intel Xeon 2.80GHz processors with total 4GB main memory. The algorithms were implemented in C++. Following the common practice in the literature, we used independent, correlated and anti-correlated databases proposed in [1] as benchmark databases. We experimented with datasets having cardinality in the range of [100k, 1M] and dimensionality(DIM) in the range of [3,7]. Number of distinct values per dimension, in other words Column Cardinality (CC), is another important parameter in our experiments as it determines the size of the bitmap table. The datasets we used for the experiments have column cardinality in the range of [20, 100]. We measure the query time for the different algorithms for skycube computation. The run time measured for our algorithms include the time taken to build the bitmap structure and the time to compute the skycube results.



(a) Varying Number of Tuples (DIM=4, CC=30) (b) Varying Dimensions (# of tuples=1M, CC=30)

Figure 1. Point vs. Value vs. TDS for correlated dataset

7.1 Comparing Point and Value algorithms

Throughout the experiments, we observed that Value-based algorithm generally outperforms the Point-based counterpart. Only exception was the case with correlated datasets. As shown in Figure 1, with correlated dataset, the Point algorithm outperformed the Value algorithm (and TDS) moderately in varying number of tuples test (Figure 1(a)) and somewhat significantly in varying dimensions test (Figure 1(b)). In the test with varying column cardinality (graph not shown), the point algorithm performed comparably with the other two algorithms.

It is because the Point-based algorithm processes points in single dimensional skyline first, most points get pruned in the initial stages of the algorithm as single dimension skyline points are good in other dimensions as well (due to correlated effect). On the other hand, Point algorithm has to process much more points for independent and anti-correlated datasets and hence its performance substantially degrades in those cases. For example, in independent dataset containing 1M points having 4 dimensions and 20 distinct values per dimension, Point algorithm takes 872 seconds to calculate skycube whereas Value and TDS finish in 11 and 30 seconds respectively. Hence, for the rest of our evaluation, we only consider Value and TDS algorithms for independent and anti-correlated datasets.

7.2 Effect of Tuple Cardinality

In this experiment, we study the effect of increase in the number of tuples on the overall skycube computation time. We compare the time taken for independent and anti-correlated benchmarks with cardinality between 100k to 1M for 4 dimensions, each dimension having 30 distinct values. Figure 2 shows the results of the experiment.

From the results, we find that Value-based algorithm

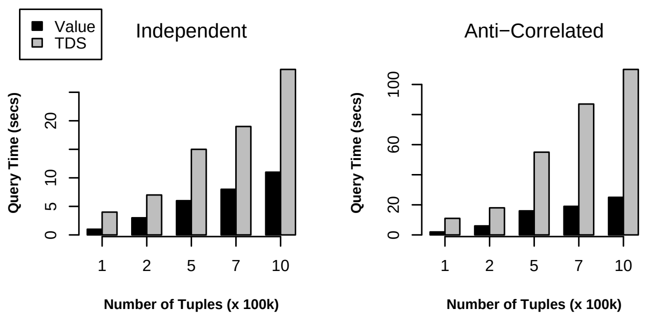


Figure 2. Effect of Cardinality (DIM=4, CC=30)

completely outperforms TDS algorithm in both datasets. Our Value-based algorithm is faster by at least by a factor of 2 for independent datasets and at least by a factor of 3 for anti-correlated datasets. It should be noted that for Value-based algorithm increase in the number of tuples increases the skycube computation time only slightly for both independent and anti-correlated databases, while there is a significant increase in computation time for TDS algorithm. As we can see in Section 7.4, time taken for the Value algorithm depends more on the column cardinality than on the number of tuples for a fixed dimension dataset.

7.3. Effect of Dimensionality

To study the effect of dimensionality on our algorithms, we use datasets having 1M tuples, 30 distinct values per dimension and vary the dimensions from 4 to 7. Experimental results are shown in Figure 3.

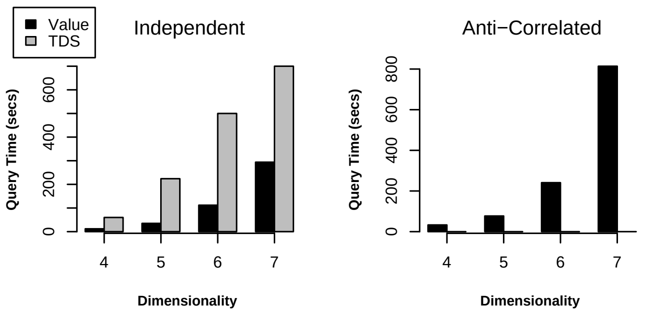


Figure 3. Effect of Dimensionality (# of tuples=1M, CC=30)[Note: TDS algorithm failed to complete for anti-correlated dataset]

It is quite clear from the figure that Value-based algorithm is more than 4 times faster than TDS algorithm for independent dataset on an average. For example, in 6 dimensions, Value algorithm completes in 112 seconds whereas TDS takes 500 seconds to run to completion. Note that the TDS algorithm failed to compute the skycube for anti-correlated dataset and only the value algorithm is shown in the second graph [3].

7.4. Effect of Column Cardinality

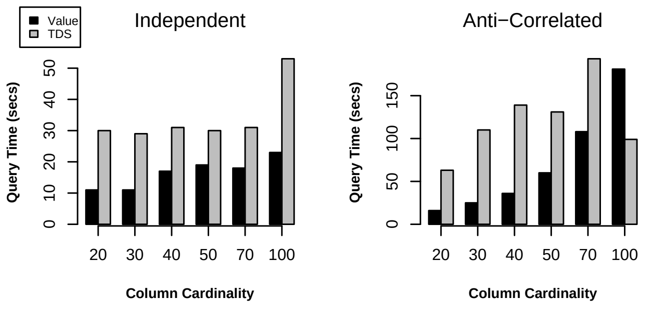


Figure 4. Effect of Colum Cardinality (# of tuples=1M, DIM=4)

Since the size of the bitmap table and hence the performance of our Value-based algorithm depend on the number of distinct values in each dimension, we study the effect of Column Cardinality on our algorithms. We considered column cardinalities in the range of [20, 100] for a dataset having 1M records and 4 dimensions. Experimental results are shown in Figure 4.

For varying dimensions, our Value-based algorithm is faster on an average by a factor of 2.1 for independent datasets and by a factor of 2.7 on an average for anti-correlated datasets. As the number of distinct values increases, search space for Value-based algorithm widens and this explains the increase in computation time. Since TDS algorithm does not depend on the column cardinality, we can expect a break even point before which Value-based algorithm would perform better and after which TDS would be preferable to our Value algorithm. In the experiment above, the break even point occurs at column cardinality of 100 for anti-correlated dataset as can be seen from the figure.

7.5. Effect of Point Pruning Strategies

In this section, we study the effect of point pruning strategies on our algorithms. We evaluated the number of bitmap accesses for the original single skyline Bitmap algorithm (explained in Section 3) and our Point-based and Value-based algorithms. The experimental results in this section are implementation independent as we measure only

[3]We only obtained the executable from the authors and were not able to investigate the problem further.

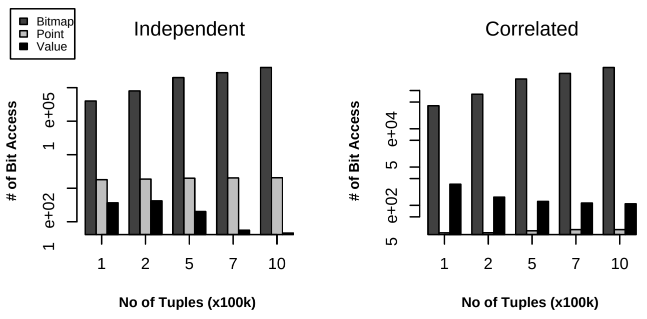


Figure 5. Efficiency of Point Pruning Strategies(DIM=4, CC=30) [Note: Graph in LOG scale]

the number of bitmap accesses and not the time taken to access them. It is important to note here that the number of bitmaps accessed by Point/Value based algorithm for calculating d dimensional skycube is same as the number of bitmap accesses done for computing single skyline of d dimensions. The bitmap accesses were measured for a dataset containing 1M records and 4 dimensions.

Results shown in Figure 5 explain the efficiency of our pruning strategies. We show the results only for independent and correlated datasets as the trend for anti-correlated dataset was very similar to independent dataset. It can be clearly seen that our skycube algorithms are orders of magnitude better than the original Bitmap algorithm. Also, as explained above, Point algorithm performs better for correlated datasets whereas Value algorithm is preferable for independent and anti-correlated datasets. It is interesting to note that the number of bitmap accesses is more or less constant for Point-based algorithm whereas it constantly reduces for Value algorithm for a particular column cardinality. This is because, for Value algorithm increasing the number of tuples also increases the probability of hitting at a skyline value combination early enough in the algorithm. This significantly reducing the search space and hence the number of bitmap accesses.

8. Conclusions

Skyline queries are important for several database applications including decision support, visualization and user preference queries. In this paper, we presented two Skycube computation algorithms that compute the skyline query results for every non empty subspace of a given set of dimensions. The first algorithm, Point-based skycube, processes dataset one point at a time to check for skyline in any dimension subset. This point-wise processing makes this algorithm preferable for user preference queries, where a given set of points, chosen by the user can be checked for skyline in any of the dimension subsets. The second algorithm, Value based algorithm, checks value combinations for prospective skyline points. It is targeted toward datasets with low column cardinalities and very high number of tuples. Our experimental study shows that our algorithms significantly outperform the current skycube algorithms for low cardinality, low dimensional datasets, while having a comparable performance in other cases. One possible direction for future work could be to come up with a hybrid algorithm that combines the advantages of both Point and Value based algorithms.

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