Traffic Grooming in Star Networks[*]

Bensong Chen, Rudra Dutta[†], George N Rouskas

Department of Computer Science,

North Carolina State University, Raleigh, NC 27695-7534

Technical Report TR–2004-16

Abstract

We study the traffic grooming problem on WDM networks with the physical topology of a star. In star networks, several nodes are connected to a single hub directly through a bi-directional optical fiber, but are not connected to each other. Previous studies concentrated on the objective of minimizing the total amount of electronic switching. However, in order to lower the network cost, we consider the objective of minimizing the number of line terminating equipments (LTE) , which has dominating cost among optical devices. We first show our study on the various complexity results of the problem. A variation of the objective to minimize the maximum number of LTEs at each node (the Min-Max objective) is also studied. Then, heuristic algorithms are given for the two objectives, respectively. Numerical results are collected to show the effectiveness of our algorithm from experiments. The relationship between the two objectives is explored as well.

1 Introduction

Wavelength division multiplexing (WDM) technology has the potential to satisfy the ever-increasing bandwidth needs of network users on a sustained basis. WDM is the process of transmitting data simultaneously at multiple carrier wavelengths over an optical fiber cable. The wavelengths can be kept sufficiently far apart so that they do not interfere with each other.

[†] Correspondence author

Thus, a single strand of fiber can be thought of as a collection of high capacity *virtual fibers*. Today, WDM systems are widely deployed in long-haul networks, and have a major presence in the metro-area networks as well.

A *lightpath* is defined as a clear channel (or *wavelength* in this case) in which the signal remains in optical form throughout the physical path between two end nodes. The set of lightpaths defines a *logical topology*, which can be designed to optimize some performance measure for a given set of traffic demands. The logical topology design problem has been studied extensively in the literature. Typically, the traffic demands have been expressed in terms of whole lightpaths, while the metric of interest has been the number of wavelengths, the congestion (maximum traffic flowing over any link), or a combination of the two. In WDM networks, nodes are equipped with *optical cross-connects* (OXCs), devices which can optically switch wavelengths, thus making it possible to establish *lightpath* connections between pairs of network nodes. Since each network node needs to terminate data destined to the site, and initiate traffic from it to other network nodes, devices are needed to add/drop signals to/from the lightpaths, and switch them to other channels, if necessary. Since user data are expressed in electronic signal, some kind of transformation between optical and electronic signals needs to be done. The OEO (opto-electro-optical) transformation is done by *line terminating equipments* (LTEs) at each network node. Digital cross-connect (DXC) can further be used to switch the electronic signals for rearranging data multiplexing onto the wavelengths.

With the deployment of commercial WDM systems, it has become apparent that the cost of network components, especially LTE, is one of the dominant costs in building optical networks, and is a more meaningful metric to optimize than, say, the number of wavelengths (since with today's technology, more than a hundred wavelengths can be multiplexed in one fiber). Furthermore, with currently available optical technology, the data rate of each wavelength is on the order of 2.5-10 Gbps, while 40 Gbps rates are becoming commercially available. In order to utilize bandwidth more effectively, new models of optical networks allow several independent traffic streams to *share* the capacity of a lightpath. These observations give rise to the concept of *traffic grooming*, a variant of logical topology design, which is concerned with the development

of techniques for combining lower speed components onto wavelengths in order to minimize network cost.

In this paper, we consider the traffic grooming problem in the star topology. Star networks arise in the interconnection of LANs or MANs with a wide area backbone. Cable TV networks and passive optical networks (PONs) are based on a tree topology, which can be decompose into stars as well. Consider a relatively small optical WDM network with a general topology. If we require that only one of the network nodes has switching ability, the virtual topology formed by a traffic grooming solution will be exactly like that in the star network. Although the direct lightpaths that 'pass through' the hub node may not actually pass the OXC at the hub, the virtual topology (by ignoring the physical links) will look just like that in the star topology. Therefore, if we use the star virtual topology as building block, we can solve larger and more general network topologies by a good decomposition method.

Typically, the goal of existing traffic grooming studies is to minimize the LTE cost, given a set of traffic demands. To this end, the problem is formulated so that its objective reflects the amount of LTE either directly (e.g., by counting the number of lightpaths established) or indirectly (e.g., by counting the amount of electronic – as opposed to optical – routing performed). However, most studies concentrate on some *aggregate* representation of the LTE cost. That is, the objective to be minimized is usually expressed as the sum, over all network nodes, of the LTE cost at each individual node. While a metric that accounts for the network-wide LTE cost is important, minimizing the total cost in the network without imposing any bound on the cost of individual nodes may result in a solution in which some nodes (such as the hub node in the Star topology) end up with a (very) large amount of LTE while some others with only a small amount of LTE. Such a solution may have a number of undesirable properties. First, a node that requires a large amount of LTE may be too expensive or even impractical to deploy (e.g., due to high interconnection costs, high power consumption, or space requirements). Second, the resulting network can be highly heterogeneous in terms of the capabilities of individual nodes, making it difficult to operate and manage. Third, and more important, a solution minimizing the total LTE cost can be extremely sensitive to the assumptions regarding the traffic pattern,

as previous studies [10] have demonstrated. Specifically, a solution that is optimal for a given set of traffic demands may be far away from optimal for a different such set. Since LTE involve expensive hardware devices that are difficult to move from one node to another on demand, an approach that attempts to minimize total LTE cost may not be appropriate for dimensioning a network unless the network operator has a clear picture of traffic demands far into the future *and* these traffic demands are unlikely to change substantially over the life of the network.

We also note that a similar approach of minimizing the maximum nodal cost was taken in [3] in a different context, namely for routing and wavelength assignment in the presence of converters. The algorithm was used in our study of traffic grooming in Ring networks in [2], which concentrated on the approach of minimizing the maximum nodal LTE cost.

Because of its importance and difficulty, the traffic grooming problem has drawn a lot of attention in recent years. In [9], the authors gave a survey and classification of relevant work on the *virtual topology design* problem on or before year 1999. Some surveys on the general *traffic grooming* problem can be found in [11, 20, 5, 24].

Recent studies aimed at more general mesh topologies can be found in [8, 18, 23]. More studies with the objectives of minimizing LTE cost can be found in [16, 17, 19, 13].

In [22], *dynamic grooming* is considered in which efficient reconfiguration problem is studied using a novel graph representation of the problem. The grooming problem is then transformed into corresponding problems in graph theory. The Min-Max objective we consider in this research assumes balanced equipment capability at each network node, which offers more room for change of traffic patterns in dynamic grooming.

The grooming problem in the star topology has also gained interest in recent researches. The objective considered in these papers is to minimize total amount of electronic switching (thus the delay introduced by OEO transformation), which is related to, but different from, the objective considered in this study.

In [4], the authors first proved that the problem is equivalent to a Maximal Weighted Local Constraint Subgraph (MWLCS) problem, which can be proved to be NP-Complete. Then, they gave a greedy algorithm that guarantees a 2-optimal solution.

In [1], the authors considered two versions of the problem: minimizing the electronic solution and maximizing the optical switching. Besides proving NP-Completeness, they also proved that approximation approach on one version cannot be transformed to approximation on the other, and gave approximation approaches for both versions separately, by transforming the corresponding versions of the problem to existing NP-Complete problems. A polynimial-time optimization algorithm is also given for the special case in which only two wavelengths are available on each fiber.

Also, there is similar study in [7]. The authors studied the grooming problem in elemental network topologies, including stars. For star networks, they gave complexity proofs, and by pruning the search tree, found a method that can give a series of upper and lower bounds. A greedy heuristic was provided to make improvement towards the objective at each iteration. The paper inspired ideas for similar NP-Complete proofs in our paper.

In this paper, we concentrate on LTE cost. This is an important measure of the actual cost of a network. We consider both the problems of minimizing this cost as a total over all the network nodes, as well as minimizing the LTE cost at the node where this cost is maximum (the min-max problem). We discuss the relation between the two, and present some intuitive insights in this regard, as well as providing heuristic algorithms for each objective that perfrom well in practice.

The report is organized as follows. In the next Section we define the problem precisely. Section 3 presents our results on the computational complexity of the various star network grooming problems. In Section 4 we present our heuristic algorithms. The next few sections present results of numerical results and discussions pertaining to them. Section 8 concludes the report.

2   Problem Definition

In this part, we first review the well-known *routing and wavelength assignment* (RWA) problem and results from previous studies, and then introduce the traffic grooming problem, and the problem restricted to the star topology.

2.1 The RWA Problem

An optical network can be abstracted as a directed graph, with vertices representing network nodes (sites), connected with directed edges showing the optical fiber links. A traffic demand from node s to node d (denoted as $t^{(sd)}$ can be carried on a certain physical route of fiber links, and by certain wavelengths on each of the links. If we consider a set of such demand T, the RWA problem will become how to carry all the traffic demands from the respective sources to the destinations using the available wavelengths. Since data carried for each demand are different, this is a *multicommodity flow* problem, so if two demands share the same fiber link, they must be carried on different wavelengths. The goal of the RWA problem is to satisfy all traffic demands in T, while minimizing the number of wavelengths used in the whole network. Generally, RWA problem assumes no use of wavelength converters in the network, that is, each traffic demand is carried on the same wavelength throughout the routing path.

Previous studies show that wavelength assignment to minimize the number of wavelengths can be solved in polynomial time in paths and stars. And it is easy to see that for the general tree topology, each pair of nodes is joined by a unique path, which means that routing is fixed in trees. For wavelength assignment on trees to minimize the total number of wavelengths, the problem is NP-hard [6].Not surprisingly, the RWA problem is NP-hard in general network topologies [15] as well. However, if the topology is a star, it is equivalent to Minimum Edge Coloring in bipartite graph, which is solvable in polynomial time [21].

2.2 The Traffic Grooming Problem

Current optical technologies allow for multiplexing lower-rate traffic streams onto the same wavelenght using time-division multiplexing. If we require that only traffic belonging to the same source/destination pair be multiplexed onto the same lightpath, it is equivalent to the RWA problem discussed in the previous section. However, this constraint means that we have to set up direct lightpaths for each source/destination pair, which is generally impractical due to wavelength constraints or optical device constraints.

For that reason, each node in the optical network needs to do both optical and electronic

switching. The network node let some lightpaths pass through with only optical switching, but terminate/originate other lightpaths. Some traffic may be switched electronically onto new lightpaths to be carried to its destination. The electronic switching is also called *grooming*, which allows for better use of wavelength capacity, reduce wavelength requirements, and enhances *virtual connectivity*. As a trade-off, expensive opto-electro-optic devices (e.g., line terminating equipment) and electronic switches (digital cross-connects) need to be equipped at the network nodes. The traffic grooming problem is thus defined for balancing the advantages and costs.

We define a positive integer C as the capacity of one wavelength, expressed as units of some basic transmission rate (such as OC-3). The capacity C is also called the *grooming factor*. Let W be the number of wavelengths that each fiber can carry cuncurrently. A traffic demand matrix $T = [t^{(sd)}]$ can be defined, where integer $t^{(sd)}$ denotes the number of basic transmission rates from node s to node d. (We allow the traffic demands to be greater than the capacity of a lightpath, i.e., it is possible that $t^{(sd)} > C$ for some s, d.) Given the traffic matrix, the traffic grooming problem involves the following conceptual subproblems (SPs): (1) *logical topology SP:* find a set R of lightpaths that forms a virtual topology, (2) *lightpath routing and wavelength assignment SP:* solve the RWA problem on R, and (3) *traffic routing SP:* route each traffic stream through the lightpaths in R.

The first and third subproblems together constitute the grooming aspect of the problem. Also, the number W of wavelengths per fiber link is taken into consideration as a constraint rather than as a parameter to be minimized.

The optimization goal we consider is to minimize the cost of LTEs in the network, which is the same as minimizing the number of lightpaths established in the system (since each lightpath requires LTE at both ends). Note that this objective is equivalent to minimizing the number of edges in the logical topology formed by lightpaths. We also consider another goal of minimizing the *maximum number of lightpaths originating from or terminating at any node.* This is the same as minimizing the *maximum nodal degree* in the logical topology.

We restrict our study to the physical topology of star in this paper. Figure 1 shows a star
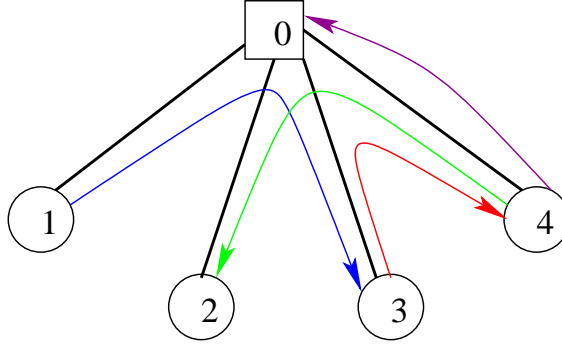
Figure 1: A 5-node star with 4 lightpaths

network with 5 nodes. We always label the central hub node with 0, followed by non-hub nodes. We assume that every physical link is bi-directional, and no traffic demand can traverse the same physical link more than once. This assumption can ensure efficient use of wavelength capacity. Thus, only the hub node can be the intermediate stop for a traffic component, while non-hub nodes are not allowed to switch traffic. The assumption will affect the nonzero variables allowed in the ILP formulation, as can be found in [12].

Under the assumptions, there will be only two kinds of lightpaths in the logical topology. The first kind consists of single-hop lightpaths which either originate at a non-hub node and terminate at the hub node, or vice versa. The second kind consists of two-hop lightpaths that originate and terminate at non-hub nodes, and are switched optically at the hub node.

In star networks, wavelength assignment can always be done in polynomial time, as long as other wavelength constraints are followed [12]. However, we prove in the next section that the whole grooming problem remains intractable.

3   Complexity Results for Stars

As we mentioned, it has been known that the wavelength assignment subproblem can be solved in polynimial time for star networks [12]. However, we are able to prove that the whole grooming problem in stars, no matter if bifurcated routing is allowed or not, still remains NP-Complete.

We consider two types of traffic routing, the case where bifurcated routing of traffic is allowed and not allowed. Specifically, for any source-destination pair (s, d) such that $t^{(sd)} \leq C$,

we require that all $t^{(sd)}$ traffic units be carried on the *same* sequence of lightpaths from source s to destination d. On the other hand, if $t^{(sd)} > C$, it is not possible to carry all the traffic on the same lightpath. In this case, we allow the traffic demand to be split into $\lfloor \frac{t^{(sd)}}{C} \rfloor$ subcomponents of magnitude C and at most one subcomponent of magnitude less than C, and the no-bifurcation requirement applies to each subcomponent independently.

For bifurcated routing, we have the following conclusion:

Theorem 3.1 *The decision version of the grooming problem in star networks with the Min-Max objective (bifurcated routing of traffic allowed) is NP-complete.*

Proof. The proof is appended as Appendix 1.

If we use the same construction as in the proof, but set the objective to be the Overall number of lightpaths in the virtual topology, we can apply the same reduction to prove on the Overall objective as well. Then we can claim the following corollary:

Corollary 3.1 *The decision version of the grooming problem in star networks with the Overall objective (bifurcated routing of traffic allowed) is NP-complete.*

Similarly, for the non-bifurcated case, we have:

Theorem 3.2 *The decision version of the grooming problem in star networks with the Min-Max objective (bifurcated routing of traffic NOT allowed) is NP-complete.*

Proof. The proof is appended as Appendix 2.

Again, the problem restriction allows us to extend the result to the Overall objective as well. Because of the construction in the proof, we can get the following aggregated result for the non-bifurcated case in the star topology:

Corollary 3.2 *The decision version of the grooming problem in star networks with the Min-Max or the Overall objective (bifurcated routing of traffic NOT allowed) is NP-complete, even when a logical topology is provided.*

9

4 Heuristic Grooming Algorithm for Stars

Now that we have proved NP-Completeness in both cases, we will concentrate our study on the situation in which bifurcated routing is allowed. This is because it can give better solution with respect to our objectives, and some security issues will favor the bifurcated case as well.

Clearly, a polynomial-time algorithm is needed for large-sized star networks to give near-optimal grooming solution. In this section, we present our polynomial-time algorithm on grooming stars for the Min-Max objective and the Overall objective, respectively.

4.1 Star Grooming for the Min-Max Objective

As we have mentioned before, wavelength assignment can be done in polynomial time, so we will concentrate on the virtual topology and routing problem in our heuristic algorithm.

The detailed algorithm is described in Figure 2.

The concept of *reduction* of a traffic matrix is to reduce the matrix T so that all elements are less than the capacity C of a single wavelength, by assigning a whole lightpath to traffic between a given source-destination pair that can fill it up completely. The available wavelengths on the links of the path segment from the source to the destination node are also decremented by the number of lightpaths thus assigned. Since breaking such lightpaths would increase the amount of LTE at some intermediate nodes of the path, this procedure does not preclude us from reaching an optimal solution, nor does it make the problem inherently easier or more difficult. We continue using the same notation for the traffic matrix and traffic components, but in what follows they stand for the same quantities after the reduction process.

The step for feasibility check is very simple: just examine if the all-electronic solution violates any constraint described in the ILP formulation in Section 2.

Note that after getting the all-electronic solution, we have reached the lower bounds for the degrees at non-hub nodes. Since we are considering the Min-Max objective, if the hub degree is even smaller than one of the non-hub degrees, we have already reached the optimal; otherwise, we need to lower the hub degrees without increasing our Min-Max objective.

The complexity of the algorithm is easily seen: the reduction and initial feasible solution

---

Min-Max Traffic Grooming Algorithm for Star Networks

Input: A star network with N non-hub nodes and a hub 0, W wavelengths, capacity C of each wavelength, and traffic matrix $T = [t^{(sd)}]$.

Output: The number of lightpaths b_{ij} from node i to node j of the star, and traffic routing quantities $t_{ij}^{(sd)}$, so that the solution is feasible and has a small value of the min-max degree objective.

procedure **StarMinmaxGrooming**

1. Reduce the traffic matrix by assigning direct lightpaths of capacity C fully used

2. Use single-hop lightpaths to carry the remaining two-hop traffic

3. Check feasibility. If infeasible, exit

4. Initialize $b_{ij}, t_{ij}^{(sd)}$, record indegree I_j, outdegree O_j and remaining capacity r_{ij} on lightpath (i, j) in the current topology

5. $u \leftarrow$ max degree of the non-hub nodes

6. **while** *max hub degree* $> u$ **do**

7.   Sort all the two hop residual $t^{(sd)}$ in non-incresing order

8.   **for each** of the sorted $t^{(sd)}$ **do**

9.   **if** carrying the traffic directly doesn't increase u **then**

           re-route the traffic on direct lightpaths, update all variables accordingly

10.   **endfor**

11.   **if** $u < w$ and *max hub degree* $> u$ **then** $u + +$

12.   **else** break; **endif**

13. **endwhile**

14. Use the polynomial-time WLA algorithm in [12] to assign wavelengths to the lightpaths

end **Procedure**;

Figure 2: Traffic Grooming Algorithm for Star Networks with the Min-Max Objective

takes $O(N^2W)$; the *while* loop between Steps 6-13 takes no more than W times, since a feasible solution requires $u \leq W$; and there are $N(N-1)$ traffic elements to consider within the loop. Thus, the overall complexity of the algorithm is $O(N^2W)$.

4.2 Star Grooming for the Overall Objective

First, we argue that the Overall results given by our Min-Max heuristic in Figure 2 can hardly be improved without violating the Min-Max objective. This is because we aimed at the Min-Max objective alone in the algorithm. We had very good results: for stars with 10 non-hub nodes, only 1 out of the 50 cases gives non-optimal solution. And that is at the expense of neglecting the Overall objective. Let us consider the following approaches: (1) Continue rerouting two-hop traffic onto direct lightpaths. This cannot be done without violating the existing Min-Max solution, because at the last iteration of the *while* loop at Step 6-13, we have rerouted all such traffic that will not violate the current objective u in nonincreasing order. The *if* statement at Step 9 ensures that. (2) Breaking some direct lightpaths back into two at the hub node. This can only be done by examining the last iteration of the *while* loop step by step, considering the candidate traffic in nondecreasing order (because smaller traffic are more likely to be fit into remaining capacities). Moreover, we need to ensure that (a) the hub degree cannot increase beyond u, and (b) the non-hub degree cannot increase beyond u. Note that at the previous iteration, we have reached the limit that the non-hub degrees cannot increase beyond $u-1$, so there is little room left for improvement. Finding a good combination of the above two approaches will fall into combinatorial explosion.

Therefore, if we want to balance between the two objectives, new approaches are needed. To find a good algorithm, we first examine the Integer Linear Programming (ILP) formulation for the Star grooming problem.. A more general (and more straightforward) ILP can be found in [12], but here we give a simplified version that has only binary (0-1) variables. The formulations were similar to the ones in [1], but with changes to consider the new objective, and relaxing the constraints to allow for traffic demands between each source/destination pair to exceed the wavelength capacity C. Corresponding definitions can be found in Section 2.2.

We need to define the status after we make the reduction (by setting up full direct lightpaths and one-hop lightpaths for neighboring s/d pairs). Suppose that after the reduction, we get:

$t_r^{(sd)}$: the traffic demand ($0 \leq t_r^{(sd)} < C$) from s to d, $s \neq d \neq 0$. We further define $t_{out}^s = \sum_d t_r^{(sd)}, t_{in}^d = \sum_s t_r^{(sd)}$.

r_{out}^s: the remaining capacity left on the (possibly) underutilized lightpath $(s, 0)$. Similarly, r_{in}^d can be defined.

w_{out}^s: the full wavelengths available on link $(s, 0)$. Similarly, w_{in}^d can be defined.

We need to find $x^{(sd)} \in \{0, 1\}$, in which 0 denotes electronic routing of remaining demand $t_r^{(sd)}$, while 1 denotes optical routing (setting up a two-hop lightpath dedicated to it).

Therefore, our goal to minimize the total number of lightpaths can be expressed as the following. We don't count the lightpaths set up during reduction, because they are necessary and we have no choice but to keep them. The new goal is to minimize:

$$\sum_s \left\lceil \frac{t_{out}^s - r_{out}^s - \sum_d t_r^{(sd)} x^{(sd)}}{C} \right\rceil + \sum_d \left\lceil \frac{t_{in}^d - r_{in}^d - \sum_s t_r^{(sd)} x^{(sd)}}{C} \right\rceil + \sum_{s,d} x^{(sd)} \tag{1}$$

Subject to (link capacity constraints):

$$t_{out}^s - r_{out}^s - \sum_d t_r^{(sd)} x^{(sd)} + C \sum_d x^{(sd)} \leq C w_{out}^s, \forall s \tag{2}$$

$$t_{in}^d - r_{in}^d - \sum_s t_r^{(sd)} x^{(sd)} + C \sum_s x^{(sd)} \leq C w_{in}^d, \forall d \tag{3}$$

Note that except for $x^{(sd)} \in \{0, 1\}$, all other values can be calculated from the traffic matrix very easily, so the total number of variables are at most $N(N - 1)$ each in binary mode, with the solution space of $2^{N(N-1)}$ combinations at most.

If we allow for relaxing the ceiling operations in the objective, we can get a new formulation that inspires the greedy algorithm.

The constraints are (link capacity constraints):

$$\sum_d (C - t_r^{(sd)}) x^{(sd)} \leq C w_{out}^s - t_{out}^s + r_{out}^s, \forall s \tag{4}$$

$$\sum_s (C - t_r^{(sd)})x^{(sd)} \leq Cw_{in}^d - t_{in}^d + r_{in}^d, \forall d \tag{5}$$

The goal is to minimize:

$$\sum_{s,d} (C - 2t_r^{(sd)})x^{(sd)} + \left(\sum_s (t_{out}^s - r_{out}^s) + \sum_d (t_{in}^d - r_{in}^d) \right) \tag{6}$$

The latter part is a constant. The ILP formulation resembles the *Multiconstraint 0-1 Knapsack Problem* (MKP), also called Multi-Dimensional 0-1 Knapsack Problem (MDKP). However, it has special forms that characterize the star grooming problem, so better approaches can be taken to get near-optimal solutions.

We can now analyze the algorithm using the 0-1 ILP formulation for the problem. From the goal, we know that we should try to route all traffic demands that are greater than $C/2$ optically, so that the values $(C - 2t_r^{(sd)})x^{(sd)}$ are negative, and will decrease the goal. However, we should guard against the constraints as well. Therefore, our intention is to greedily route the traffic demands that are more than $C/2$ optically, while making sure not to violate the constraints.

Note that minimizing $\sum_{s,d} (C - 2t_r^{(sd)})x^{(sd)}$ is the same as maximizing $\sum_{s,d} (2t_r^{(sd)} - C)x^{(sd)}$. So, for the relaxed objective, at each iteration, we are closer to our goal by $2t_r^{(sd)} - C$ by routing $t_r^{(sd)}$ directly onto one lightpath. However, for the original objective with the ceiling operations, the performance of each iteration will depend on the remaining capacities on the two corresponding lightpaths $(s, 0)$ and $(0, d)$. For instance, even if a $t_r^{(sd)} < C/2$, when both lightpaths have remaining capacity $\geq C - t_r^{(sd)}$, routing the traffic directly on a single lightpath will decrease the total degree by 2; accordingly, some traffic greater than $C/2$ may actually introduce a new lightpath without eliminating either of the two one-hop lightpaths, adding penalty to the objective. Therefore, we can make potential improvement by trying to route more traffic directly which may $\leq C/2$, and if the results are better, we should definitely accept the better solution.

In the algorithm for the Min-Max objective, what we typically do is to start from an all-electronic solution, in which the hub degree is generally the highest, then try to lower the hub

14

degrees while watching the growth of the maximum non-hub degree, until the hub degree is no greater than the maximum of degrees at other nodes.

If our goal is to minimize the overall objective, several changes need to be made to the algorithm. Instead of rewriting all the steps here, we will just describe the algorithm without details. The basic approach is to greedily create two-hop traffic demands with direct lightpaths.

- **Step 1.** Find the all-electronic solution as the base. Record the variables and the overall degree u_0.

- **Step 2.** Sort all the two hop residual $t^{(sd)}$ in non-incresing order, labeled with $1, 2, \ldots, N^2$.

- **Step 3.** With the sorted list, try to carry the first traffic demand on the list directly on a new two-hop lightpath, and update the corresponding variables and the virtual topology. Record the resulting overall degree as u_1.

- **Step 4.** Repeat Step 3 with the remaining sorted list, until the traffic demand value reaches 0. Record the overall degrees at each repetition. Finally we will get a series of numbers $u_0, u_1, \ldots, u_m, m \leq N^2$.

- **Step 5.** Find the smallest value of u in the list, and use the corresponding virtual topology as the solution.

The idea of the above algorithm comes from the fact that the optimal overall solution must lie between the all-electronic and the all-optical solutions. Furthermore, intuitively, carrying larger traffic demands on two-hop lightpaths will make better use of the wavelength capacity. The disadvantage of the all-electronic solution is excessive hub degrees, while the drawback of the all-optical solution is the many underutilized lightpaths that could have been saved by grooming. With our greedy approach in between, we are likely to find near-optimal results.

The complexity of the algorithm is again easy to see. Up to N^2 iterations can be made in Step 4, each can be done in $O(1)$; Finding the smallest value in a list of N^2 values takes $O(\log N)$. Thus, the overall complexity for the simple greedy algorithm is only $O(N^2 \log N)$.

5 Numerical Results for the Star Grooming Problem

In this section, we present our experimental results with stars for the two algorithms we discussed in the previous section.

the traffic matrix $T = [t^{(sd)}]$ of each of the 50 problem instances is generated by drawing $N(N-1)$ random numbers (rounded to the nearest integer) from a Gaussian distribution with a given mean t and standard deviation σ that depend on the traffic pattern. We will concentrate on the performance of our algorithms on a *random* traffic pattern. To generate a traffic matrix of that pattern, we first determine the mean value t of the Gaussian distribution according to the desired link load L, and we let the standard deviation be 150% of the mean t. Consequently, the traffic elements $t^{(sd)}$ take values in a wide range around the mean, and the loads of individual links also vary widely. With such a high standard deviation, the random number generator may return a negative value for some traffic element; in this case, we set the corresponding $t^{(sd)}$ value to zero. Also, if a traffic matrix generated in this manner is infeasible (i.e., the load on some link exceeds the value WC), then we discard it and we generate a new matrix for the corresponding problem instance. Note that with the high variation, the link load L is also not as balanced, and we can only give a range of link loads for the instances.

We will also experiment on a *uniform* traffic pattern with high loads later in our study. This is done by making changes of the Gaussian distribution parameter, to make the standard deviation to be 10% of the mean. In this case, when all links have high loads of traffic, the wavelength constraints will not allow for an all-optical solution, and the performance of the algorithms may change accordingly. The discussions can be found in Section 7.2.

5.1 Results for the Min-Max Objective

Using CPLEX with the ILP formulations, we can get some good results for the Random pattern with the Min-Max objective. Using Sun UltraSparc system, we can get optimal results For stars with $N = 16$ non-hub nodes.

After getting the initial all-electronic feasible solution, we can get a lower bound (the max degree at non-hub nodes) and an upper bound (the max degree at the hub node, if it's larger
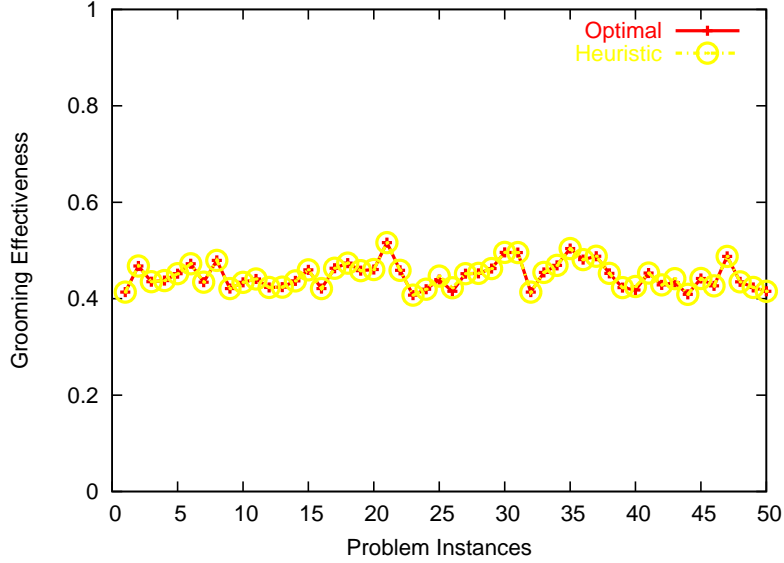
Figure 3: Star Random, $N = 16, C = 12, W = 64$, All-electronic = 119-150

than the lower bound) for our Min-Max objective. The upper bounds are too big to be plotted, so only a range is given in general.

We collected data for 50 cases with $N = 16$ and 24 each. We also used the all-electronic solution as the scale to evaluate our solutions and the optimal. A 'grooming effectiveness' is used in the evaluation, which is defined as the objective divided by the all-electronic result (at the hub node for this case) . The heuristic works even better than we have predicted. For $N = 16$, 4 of the 50 cases give non-optimal result; for $N = 24$, 18 of them are not optimal.

Figures 3 and 4 show the results respectively. We find that all non-optimal results are very close to the optimal. We also notice that the grooming effectiveness is smaller(better) when the star size grows. This result suggests that grooming techniques are more helpful in larger networks, even if we restrict the network to the simple topology of stars.

5.2   Results for the Overall Objective

The Overall algorithm, after applying all the constraints, also works pretty well for star with 10 non-hub nodes. When the star size grows to 16, most of the cases we generate cannot be solved within hours by CPLEX. The plot shows an average of 446 lightpaths for each case, and
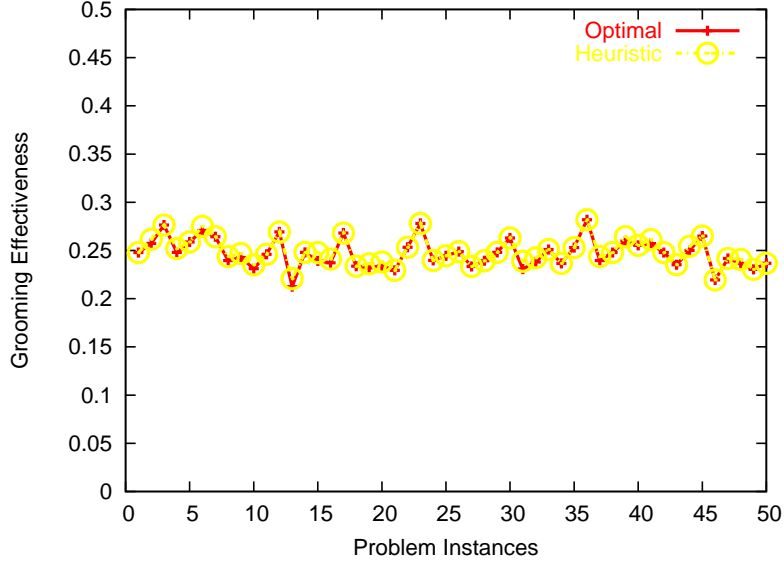
17

Figure 4: Star Random, $N = 24, C = 12, W = 64$, All-electronic = 225-259

our algorithm gives results with 2 to 4 more lightpaths. The average difference is 2.96, which is less than one percent from the optimal values.

We compare the results from this constrained case with the unconstrained case, in which there is no limit on the number of available wavelengths in each fiber. Experiments show that only 5 of the 50 cases require one more lightpath in the constrained case. This is because in the random traffic pattern with medium load, the number of wavelengths available is generally sufficient to meet the needs for the optimal solution.

6   Relationships between the Two Objectives

From the nature of strong symmetry in stars, we have the intuition that for the star network topology, the Min-Max and the Overall objective are closely related to each other. That is, by getting the Min-Max solution of the problem, the resulting overall degrees are also closed to the Overall objective, and vice versa. This is the purpose of our study in this section.

Figure 6 shows how our solutions for the Min-Max objective perform with respect to the Overall objective. We can see that for stars of size $N = 10$, our Min-Max heuristic algorithm gives 5% or less higher number of degrees than the overall optimal solutions.
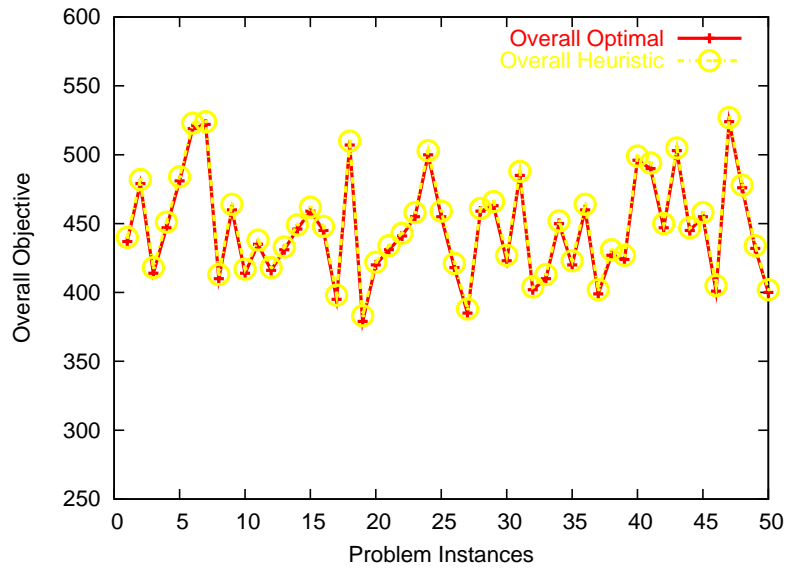
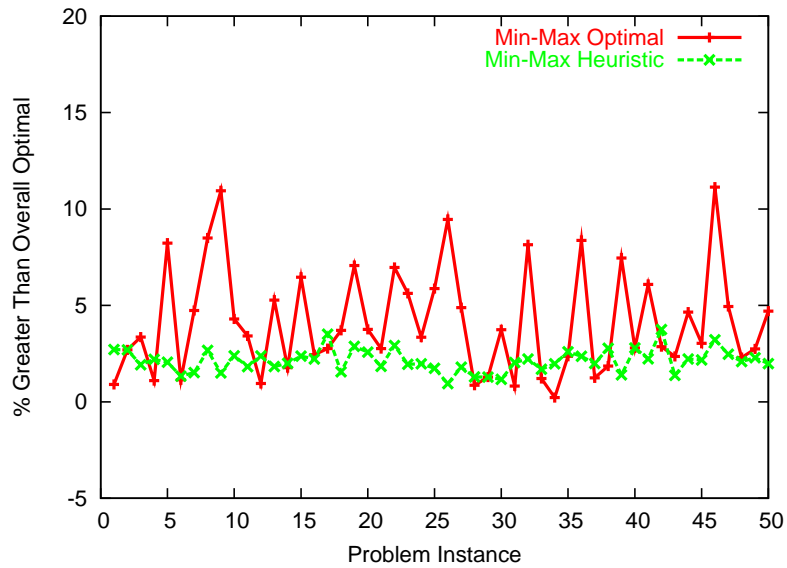Figure 5: Performance of Star Overall Algorithm, $N = 10$

Figure 6: Star Random, $N = 10$, Min-Max vs. Overall Degrees (403-528)
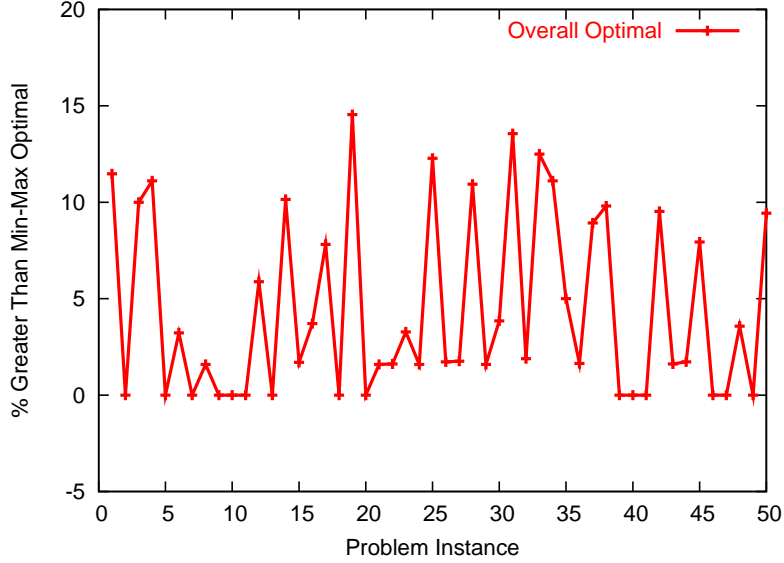
19

Figure 7: Star Random, $N = 10$, Overall Degrees vs. Min-Max (51-72)

It is interesting to notice that with respect to the Overall objective, the Min-Max heuristic algorithm gives even better results than the Min-Max optimal solutions given by CPLEX. This might be because CPLEX just return the first optimal solution from its branching-and-bound search of the solution space, and the results may have many high-degree nodes due to the search sequence.

Now we do a reverse comparison. We obtain results from the minimum Overall objective using CPLEX, find the maximum degree in the solution, then compare with the Min-Max optimal results of the same case.

The results in Figures 7 show that while we try to minimize the overall degree, the maximum degree from the solution is also not far from the optimal Min-Max objective in star networks.

7 Effect of the Greedy Algorithm on Both Objectives

In the algorithm for the Overall objective, a simple greedy approach is used without considering the Min-Max objective. We record the results for both objectives at each iteration, and find the trends from all-electronic to all-optical solutions to study the trend on both objectives. If our goal is to find good solutions that are not far away from either of the two objectives, we can

analyze the two figures showing the steps from the Overall algorithm, and find a point close to the *trough* of both.

A star of size $N = 10$ means that the number of outgoing lightpaths from s has a variation of at most 9 from all-electronic to all-optical solutions. Therefore, low traffic load with Uniform pattern means no wavelength constraint at all. For the Random traffic pattern, statistically, only a few cases will happen to reach the bounds. Experiments on 50% load show 4 out of 5 cases in which the constrained solution needs one more lightpath.

Note that after reduction, the residual traffic demand matrix will be generally random. Therefore, traffic patterns only affect the link loads (constraints). Considering also the fact that nodal degree has at most variation of $N - 1$, it can be deducted that the constraints are the main concern only when many links have very high loads concurrently.

7.1 Random Traffic Pattern with Low Loads

From our experiments, we find that the Min-Max objective will generally go down and then later up during the transformation from all-electronic to all-optical solutions. The Overall objective has the same general trend, but may 'thrash' up and down not that smoothly. As a consequence, we should not stop trying after the objective goes up for the first time. Instead, we should pick the best value from the results of all iterations.

The two examples we include here are from the *random* traffic pattern with 50% load. Figure 8 reaches best results for both objectives simultaneously around the 40th iteration. However, for the case in Figure 9, we may have to decide which objective to favor in order to choose a 'good solution'.

7.2 Uniform Traffic Pattern with High Loads

This is the case in which all physical links have high loads (95%). Five cases are studied to show differences between constrained and unconstrained cases. Interestingly, the constraints didn't affect the results a lot. And in some cases, the constrained solution even gives better results. Of the 50 cases, 2 need two more lightpaths, 7 give one more total lightpath, 33 give
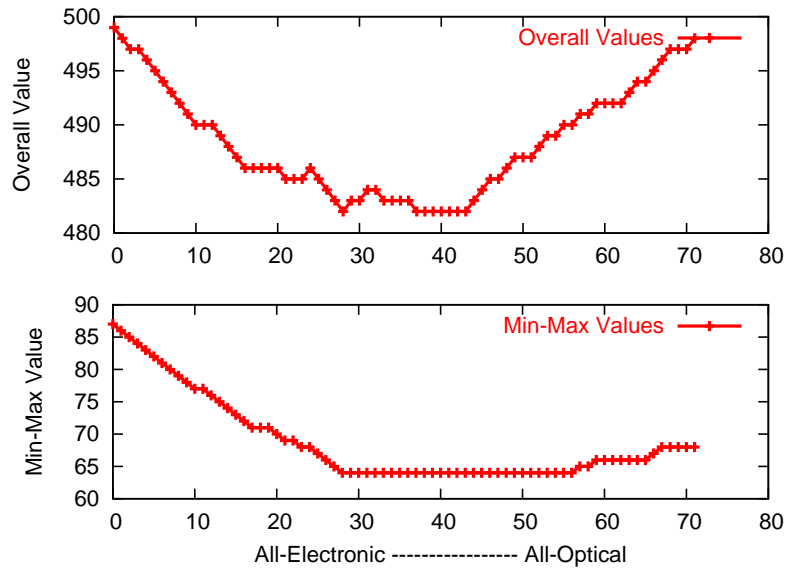
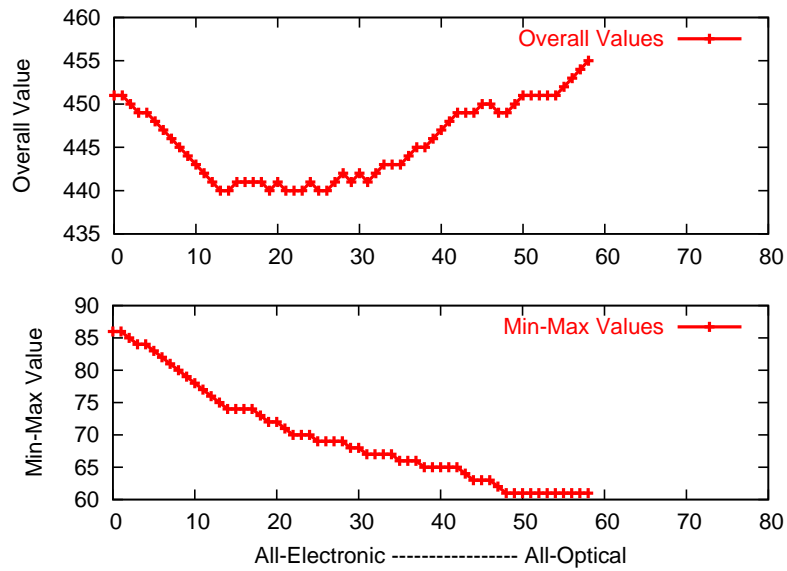Figure 8: Example 2, OPT[63, 479], Min-Max[63], Overall[64, 482]

Figure 9: Example 1, OPT[61, 437], Min-Max[61], Overall[61, 449 or 69, 440]

the same result, and 8 need one less lightpath.

In Figure 10, we record the results for each objective after one iteration. If at a certain iteration, rerouting the traffic optically will result in wavelength violation, we skip the element and leave the results for corresponding iteration blank. For instance, there is a big gap between Iterations 78 and 84, which means that the elements considered in those iterations will generate infeasible solution.

Figure 11 shows the results after relaxing the wavelength constraints. Note that this have similar effect as lowering the traffic loads in the network to, say, 75%. We find that the relaxed case will bounce up a lot in the overall objective as we iterate towards the all-optical solution. This also happens, though less significantly, in the Min-Max objective.
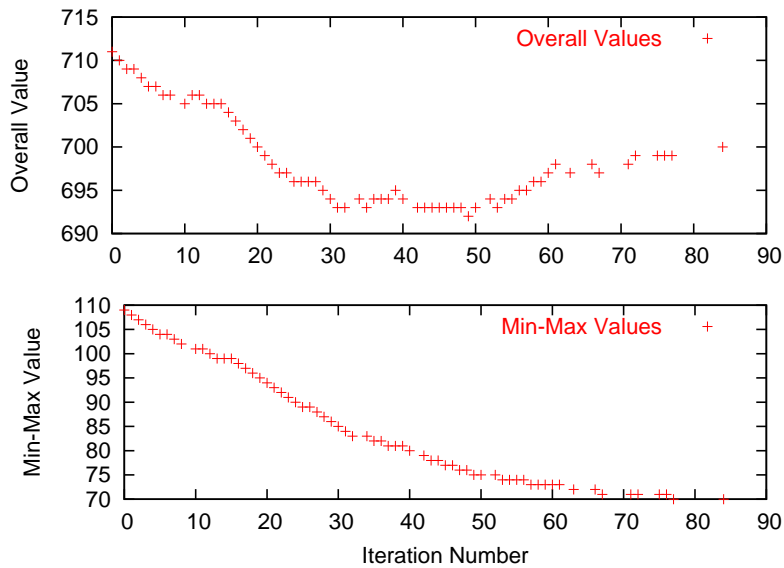

Figure 10: Example 4, Min-Max 70, Overall 692

8 Conclusions

We considered the traffic grooming problem in WDM star networks. Two objectives related to minimizing the number of Line Terminating Equipment are considered. We proved that for both objectives, the problem is NP-Complete. We proposed polynomial-time algorithms for
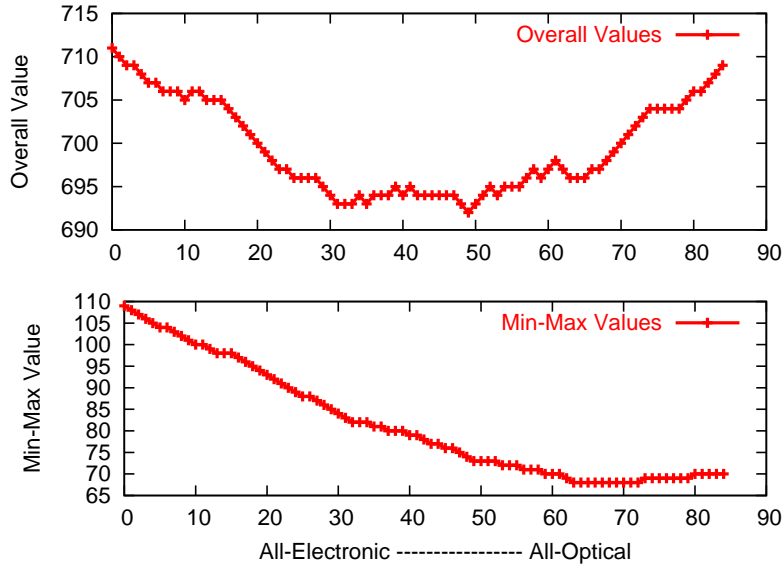
23

Figure 11: Example 4 without constraints, Min-Max 68, Overall 692

both cases, and tested their performance. We also studied the relationship of the two objectives.

Our study on star topology gives solution to WDM star networks to minimize network cost. The research can be extended to cope with more complicated network topologies with only one switching node. In this case, the virtual topology will be like a star. For larger-sized netowrks, decomposition approaches can be applied to break them into small pieces, and use the star approach to solve each piece.

References

[1] Eric Angel, Evripidis Bampis, and Fanny Pascual. Traffic grooming in a passive star WDM network. *SIROCCO 2004*, June 2004.

[2] Bensong Chen, George N. Rouskas, and Rudra Dutta. Traffic grooming in WDM ring networks to minimize electronic port cost. In *Proceedings of Networking 2004*, May 2004.

[3] L-W. Chen and E. Modiano. Efficient routing and wavelength assignment for reconfigurable WDM networks with wavelength converters. In *Proceedings of IEEE INFOCOM*, 2003.

[4] Hongsik Choi, Nikhil Garg, and Hyeong-Ah Choi. Minimal delay traffic grooming for WDM star networks. *Proceedings of OPTICOMM 2003*, October 2003.

[5] T. Cinkler. Traffic and λ grooming. *IEEE Network*, 17(2):16–21, March/April 2003.

[6] S. Corteel, M. Valencia-Pabon, D. Gardy, D. Barth, and D. Denise. The permutation-path coloring problem on trees. *Theoretical Computer Science*, 297:119–143, 2003.

[7] R. Dutta, S. Huang, and G. N. Rouskas. On optimal traffic grooming in elemental network topologies. In *Proceeding of Opticomm 2003*, October 2003.

[8] R. Dutta, S. Huang, and G. N. Rouskas. Traffic grooming in path, star, and tree networks: Complexity, bounds, and algorithms. In *Proceeding of ACM SIGMETRICS*, pages 298–299, June 2003.

[9] R. Dutta and G. N. Rouskas. A survey of virtual topology design algorithms for wavelength routed optical networks. *Optical Networks*, 1(1):73–89, January 2000.

[10] R. Dutta and G. N. Rouskas. On optimal traffic grooming in WDM rings. *IEEE Journal on Selected Areas in Communications*, 20(1):110–121, January 2002.

[11] R. Dutta and G. N. Rouskas. Traffic grooming in WDM networks: Past and future. *IEEE Network*, 16(6):46–56, November/December 2002.

[12] Rudra Dutta. *Virtual Topology Design for Traffic Grooming in WDM Networks*. PhD thesis, North Carolina State University, Raleigh, NC, August 2001.

[13] M. Esfandiari, S. Gloeckle, A. Zolfagheri, G. Clapp, J. Gannett, H. Kobrinski, V. Poudyal, and R. Skoog. Improved metro network design by grooming and aggregation of STS-1 demands into oc-192/oc-48 lightpaths. In *NFOEC 2003*, September 2003.

[14] M. R. Garey and D. S. Johnson. *Computers and Intractability*. W. H. Freeman and Co., New York, 1979.

[15] E. Harder, S.-K. Lee, and H.-A. Choi. On wavelength assignment in WDM optical networks. In *Proceedings of the 4th International Conference on Massively Parallel Processing Using Optical Interconnections*, Geneva, Switzerland, April 1997.

[16] J.Q.Hu and B.Leida. Traffic grooming, routing, and wavelength assignment in optical WDM mesh networks. Technical report, Manufacturing Engineering, Boston University, April 2002.

[17] Zenilton K.G. Patrocínio Jr. and Geraldo Robson Mateus. A lagrangian-based heuristic for traffic grooming in WDM optical networks. In *IEEE GLOBECOM 2003*, volume 5,1-5, pages 2767–2771, Dec 2003.

[18] V. R. Konda and T. Y. Chow. Algorithm for traffic grooming in optical networks to minimize the number of transceivers. In *IEEE Workshop on High Performance Switching and Routing*, pages 218–221, 2001.

[19] Chiewon Lee and E. K. Park. A genetic algorithm for traffic grooming in all-optical mesh networks. In *IEEE SMC 2002*, volume 7, 6-9, Oct 2002.

[20] E. Modiano and P. J. Lin. Traffic grooming in WDM networks. *IEEE Communications*, 39(7):124–129, Jul 2001.

[21] D. West. *Introduction to Graph Theory*. Prentice Hall, 1996.

[22] Hongyue Zhu, Hui Zang, Keyao Zhu, and Biswanath Mukherjee. Dynamic traffic grooming in WDM mesh networks using a novel graph model. *SPIE Optical Networks Magazine*, 4(3):65–75, May 2003.

[23] K. Zhu and B. Mukherjee. Traffic grooming in an optical WDM mesh network. *IEEE Journal on Selected Areas in Communications*, 20(1):122 –133, Jan 2002.

[24] K. Zhu and B. Mukherjee. A review of traffic grooming in WDM optical networks: Architectures and challenges. *Optical Networks*, 4(2):55–64, March/April 2003.

1 Proof for NP-Completeness in Star with Bifurcated Routing

We reduce the decision version of the constrained PARTITION problem to the grooming problem. An instance of the constrained PARTITION problem is given by a finite set A of $2n$ elements. Each element a_i has a weight $w_i \in Z^+$. The problem asks whether there exists a subset $A' \subset A$, such that $|A'| = n$ and $\sum_{a_i \in A'} w_i = \sum_{a_i \in A-A'} w_i$. That is, whether we can partition set A into two parts, each with n elements, and each having exactly half of the total weight of A.

Given such an instance, we construct a star network using the following transformation: $N = 2n+2, W = 2n, C = \sum_{a_i \in A} w_i$, and objective min-max degree $F = 2n$. Nodes $2n+1$ and $2n+2$ are *source nodes*, nodes $1 \ldots 2n$ are *destination nodes*. Hub node 0 is the center of the star, has both arriving and departing traffic, and is the only node that does switching, optically or electronically.

The traffic matrix $t^{(sd)}$ is:

$$t^{(sd)} = \begin{cases} C - w_d, & s = 0, d = 1, 2, \ldots, 2n; \\ w_d, & s = 2n+1 \text{ or } 2n+2, d = 1, 2, \ldots, 2n; \\ (n-1/2)C, & s = 2n+1 \text{ or } 2n+2, d = 0. \end{cases}$$

Figure 12 is the graph that shows the star network.

Notice that since the objective degree $F = 2n$, the amount of traffic demand from the hub node 0 to the destination nodes $1 \ldots 2n$ already requires outdegree $O_0 \geq 2n$. Therefore, the traffic demand from the two source nodes $2n+1, 2n+2$ to the destination nodes must *either* use the capacities left on the existing $2n$ lightpaths from node 0 to $1 \ldots 2n$, *or* set up two-hop lightpaths that bypass node 0. The capacities left on each of the $2n$ Lightpaths are so set up that any one, but not both, of the traffic components from the source nodes *has to* be routed optically for each destination node.

Now we examine the indegree of node 0 and the outdegrees of source nodes $2n+1$ and $2n+2$. The demand from source node $2n+1$ to node 0 requires set up of n single-hop lightpaths, so is the demand from source node $2n+2$ to node 0. This already makes the indegree of node 0, $I_0 = 2n$, and $O_{2n+1} = O_{2n+2} = n$. As we saw from previous analysis, we still need to set
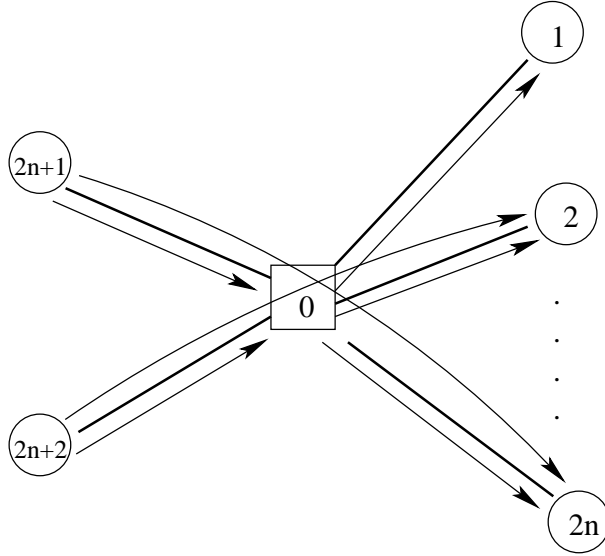
Figure 12: The Star Network with a Hub Node 0, Source Nodes $n+1$ and $n+2$, and Destination Nodes $1, 2, \ldots, 2n$. Used for Proof of Theorem 3.1

up $2n$ lightpaths that bypass node 0, either from node $2n + 1$ or $2n + 2$, for the $2n$ destination nodes accordingly. With $F = 2n$, this means that n of the lightpaths need to be from node $2n + 1$ to some n destination nodes, and another n lightpaths should be from node $2n + 2$ to the remaining n destinations. This is the same as *partitioning* set A of $2n$ elements into two sets, A' and $A - A'$, each with n elements.

Therefore, we need only to consider candidate solutions described above, which there is a two-hop lightpath from exactly one of nodes $2n+1, 2n+2$ to each destination node $1, 2, \ldots, 2n$. Then, whether the virtual topology is feasible depends on whether the remaining capacities of the single-hop lightpaths, $(2n + 1, 0)$ and $(2n + 2, 0)$, can carry the traffic that needs to be electronically switched at the hub node. We have set up the traffic demands so that the capacity left for both $(2n+1, 0)$ and $(2n+2, 0)$ is $C/2 = (\sum_{a_i \in A} w_i)/2$. As a consequence, grooming can be done *if and only if* there is a PARTITION of the $2n$ destination nodes each with n elements, and their total weights are equal. Since this constrained PARTITION problem is known to be NP-Complete [14], so is the star grooming problem. ∎

2 Proof for NP-Completeness in Star with No Bifurcated Routing

We reduce the decision version of the BIN PACKING problem to the non-bifurcated Star problem. An instance of the BIN PACKING problem has a set U of n elements, $u_1, \ldots, u_n$, having weights $w_1, \ldots, w_n \in Z^+$, respectively; a positive integer bin capacity B, and a positive integer K as the decision goal. The problem asks whether the set U can be partitioned into disjoint sets $U_1, U_2, \ldots, U_k$, such that $\sum_{u_i \in U_j} w_i \leq B, j = 1, \ldots, K$. Since the problem is trivial if there exists $w_i > B$, or $\sum_j u_j > B \times K$, or $K > n$, we can eliminate those cases, and the problem remains NP-Complete.

Given such an instance, we construct a star network using the following transformation: the star has $n + 1$ non-hub nodes, in which nodes $1, 2, \ldots, n$ are *destination nodes*, and node $n + 1$ is called the *source node*; wavelength capacity $C = B$, $W = n + K$, and the decision goal $F = n + K$.

The traffic matrix $t^{(sd)}$ is:

$$
t^{(sd)} = \begin{cases} w_d, & s = n + 1, d = 1, 2, \ldots, n; \\ nC, & s = n + 1, d = 0. \end{cases}
$$

Notice that the traffic demands from the source node to the hub node already requires the set up of n lightpaths, making the outdegree of the source node to be n. In order to reach the decision goal, the remaining traffic from $n + 1$ to the destination nodes must be 'packed' in no more than K lightpaths, each having capacity $C = B$. Since bifurcated routing is not allowed, and we've assumed $w_i \leq B$, that means each demand w_i can be packed only in one lightpath. Now, whether the degree $O_{n+1} \leq n + K$ depends on whether the BIN PACKING problem can be solved. The objective is big enough to handle degrees elsewhere, so the max degree must be O_{n+1}. Now we can say that the star problem is feasible under $F = n + K$ *if and only if* the corresponding BIN PACKING problem is decidable. Since the BIN PACKING problem is known to be NP-Complete (Please refer to [14]), so is the star grooming problem. ∎